Digitales Lernen Grundschule
Universität Potsdam

Deutsche Telekom Stiftung

Digital place value chart

Guide for teachers

Open Educational Resources

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General

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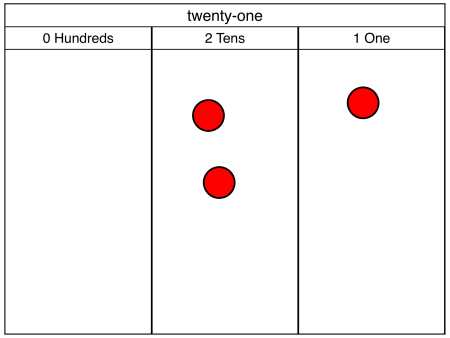
App download for iOS devices

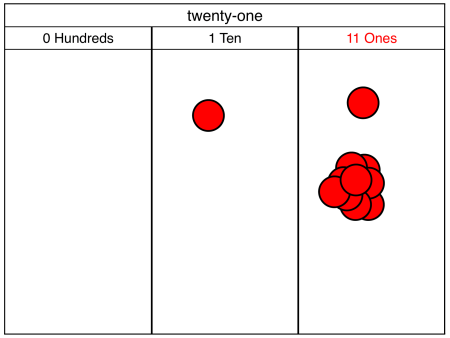
http://appsto.re/de/QfN5H.i

This guideline is used for public evaluations of the partial concept of **numerical understanding** of the project **Digitales Lernen Grundschule** at the University of Potsdam. Further information about the project, as well as contact possibilities, can be found at **http://dlgs.uni-potsdam.de**.

Basic idea of the app

The basic functionality of the app is that a number is represented by counters in the place value chart. When shifting counters, the value of the number is retained, but its representation changes.





The app is designed to accompany pupils on their way along the abstraction chain from the counting of concrete objects to the standardized symbolic representation (Gerster & Walter 1973). The focus is on teaching the place value principle. Instead of concrete objects, counters can be used whose value depends only on their position (that is their place value), and not on their color, shape or size.

The app uses virtual counters whose color is independent of their position in the place value chart. The counters can be moved on the place value chart. The value of the represented number does not change – in contrast to the non-digital place value chart – as, via automatic bundling and unbundling, the value is preserved. It can be experienced that a red counter in the tens position is the same value as 10 red counters in the ones position (Ladel & Kortenkamp 2014). It is possible to switch from a representation of the values via colors, as it is useful in the initial teaching, to representations without different colors by changing the settings of the app.

Moving a counter from one column to another column causes an automatic exchange, for example an exchange of one ten for ten ones. This basic experience can be connected: Counters can always be exchanged for counters with a lower value by moving them to the right. In this way introduction of tenths, hundredths and even smaller values is supported (Behrens & Birkner-Ahsbahs 2016).

A prerequisite for a successful use of the app is an existing understanding of the bundling principle. The app is therefore not designed for bundling and unbundling or exchanging counters with different values. For such activities, real manipulatives such as interlocking cubes or Dienes' base-10 arithmetic blocks are suitable. The app builds on such an understanding and places its focus on the impartating of a flexible understanding of the place values (see page 14).

For more information about the background of the learning environment, see
http://dlgs.uni-potsdam.de/konzepte/zahlverstaendnis.

Literature

Behrens, D. & Birkner-Ahsbahs, A. (2016). Die digitale Stellenwerttafel: Aufgabendesign zur Einführung von Dezimalbrüchen. In Beiträge zum Mathematikunterricht. (S. 117-120). Münster: WTM-Verlag.

Gerster, H.-D. & Walter, R. (1973). Mehr System im Mehrsystem-Rechnen. Zahldarstellung und Rechnen in Stellenwertsystemen. Freiburg im Breisgau: Herder.

Ladel, S. & Kortenkamp, U. (2014). „Ist das dann noch ein Zehner oder ist das dann ein Einer?“ – Zu einem flexiblen Verständnis von Stellenwerten. In Beiträge zum Mathematikunterricht. (S. 699-702). Münster: WTM-Verlag.

The app is operated through three interactions:

Adding new counters

A new counter is added to a column of the place value chart by tapping anywhere in the column.

Moving counters

When you move a counter to the right, this counter is automatically unbundled according to the place values.

When moving to the left, a counter is bundled, if possible, with further counters coming from the right.

Deleting counters

A counter can be removed by moving it upwards out of the chart.

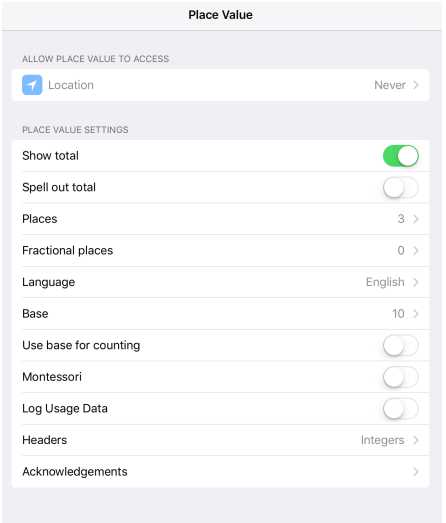
Furthermore, all the counters can be removed simultaneously by shaking the iPad.

All interactions can be performed simultaneously and independently with multiple fingers. The functionality “Gestures” should be switched off in the app settings under “General” → “Multitasking”.

For different teaching situations, you can use different display options in the app. The basic functionality of automated bundling and unbundling remains unaffected by all settings.

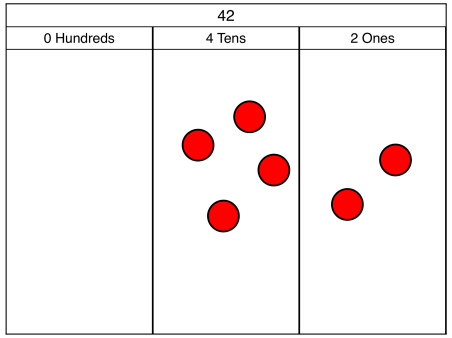
Go to the “Settings” app and select the app “Place Value” on the left side (you may have to scroll down). There you have various setting options.

Most of the setting changes cause the numbers already displayed to be deleted. You should therefore make the settings, or have them carried out by the pupils, before they work with the place value chart.

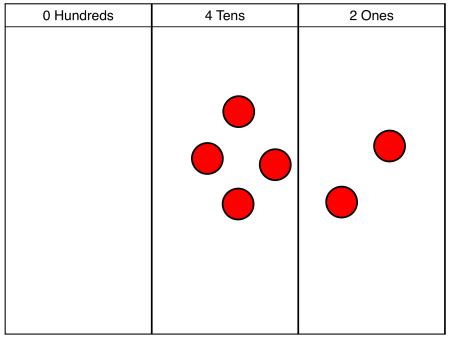


Show total

You can set whether or not the total value of the inserted counters is to be displayed in the header.



“Show total” on

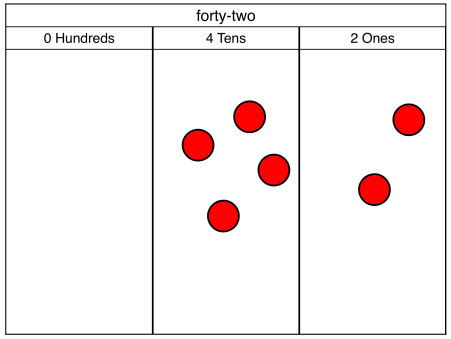


“Show total” off

Settings

Spell out

If this setting is active, the total value is displayed as a word.
Note: This setting only takes effect “Show total” is activated.



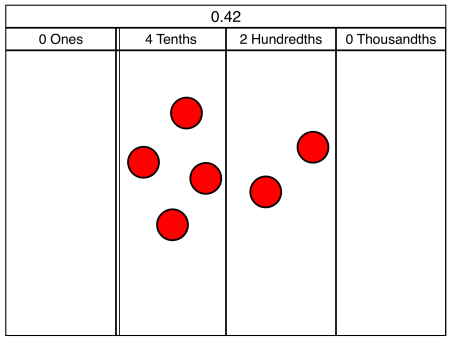
“Spell out” on

Places

Use this setting to specify the number of columns displayed.
Possible values are 2, 3 or 4

Fractional Places

If you would like to use decimal fractions, you can set the number of decimal places.
Possible values are 0, 1, 2 or 3



Places: 4, Fractional Places: 3

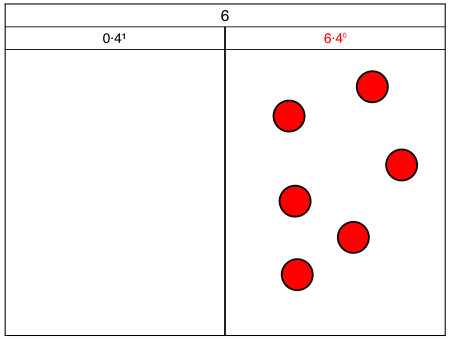
Language

The language settings influence the number words and bundling units.

The “Show total” and “Spell out” settings are especially useful for reading numbers from the place value chart as well as learning how to read the numbers. Thus, the spelling of single digits behind the comma can also be learned for decimal fractions.

Base

In addition to the decimal system (base 10), you can choose other bases here. This influences how many counters are either bundled or unbundled.
Possible values: 2 to 16



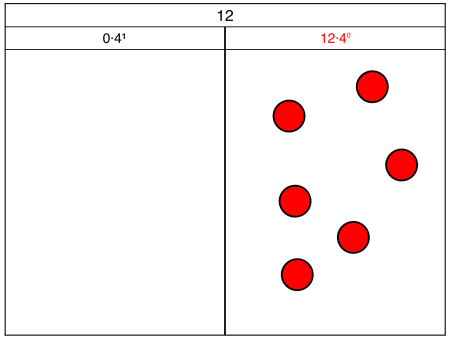
“Use Base for Counting” off

Use Base for Counting

If a numeral system other than the decimal system is set, you decide whether the automated counting is done in the decimal system or in the selected numeral system.

Montessori

Based on the colors used in Montessori materials, ones are shown as green, tens as blue, and hundreds as red.



“Use Base for Counting” on
6 in the decimal system corresponds to **12** in the base-4 numeral system
($6 = \mathbf{1} \cdot 4^1 + \mathbf{2} \cdot 4^0$)

Although the Montessori mode can be an initial aid in the identification of the place values, the mode should be avoided in the course of the primary schooling. The value of a counter is determined by its position and not by its color.

Settings

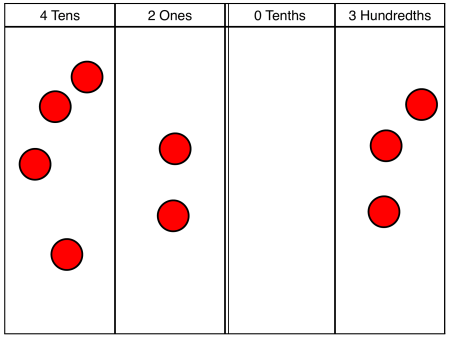
Logging

When this function is active, the movements on the iPad screen are recorded. If the iPad is connected to a computer via iTunes, corresponding recordings can be processed further. This function is intended for research purposes.

Headers

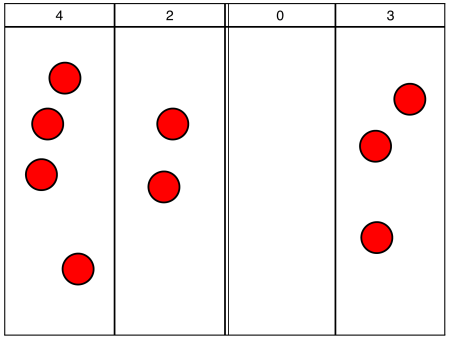
This setting affects how the place values in the header of the table are displayed and how the number of counters is counted.

Normal



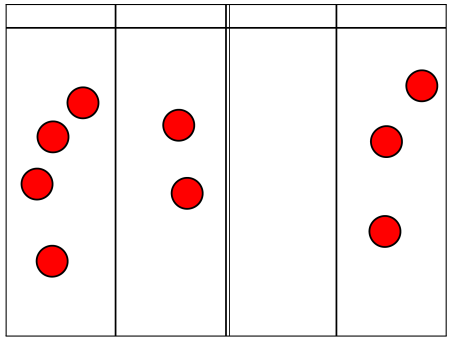
The values are written out and their quantity is counted.

Count



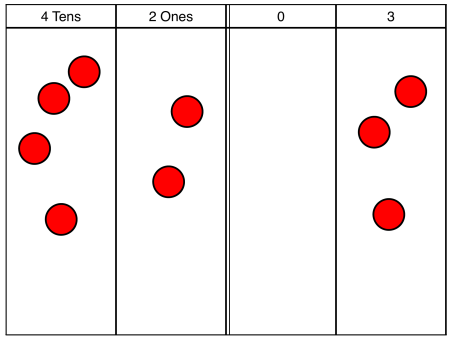
Only the quantity of counters per place value is counted.

None



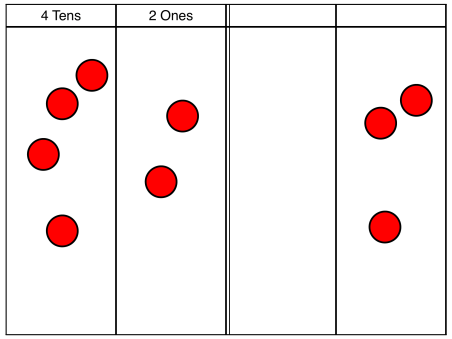
The table header is not labeled at all.

Integers & Count



In the case of the integers, the place values are written out and the counters are counted; in the case of the decimal places, only the counters are counted.

Integers



In the case of the integers, the place values are written out and the counters are counted. The decimal places are not displayed in the table header.

App Instruction

During the app introduction, the various functionalities of the app, as well as the respective mathematical background, can be conveyed through various questions. The steps presented here do not have to be introduced immediately, as they can also be carried out over several classes. Before individual functionalities are considered, a **free trial** of the app can take place, in which the pupils first act without concrete work assignments, so that pupils gain experience in using the app.

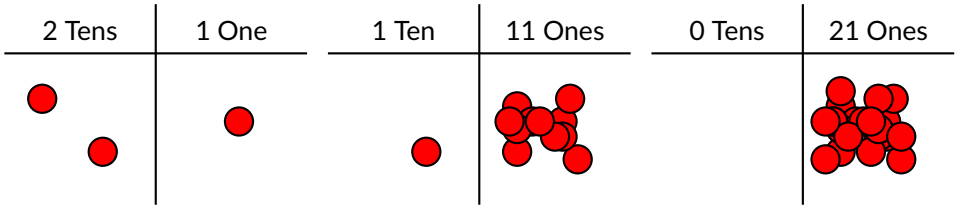
Recommended settings: Show total on, Headers Normal

Functionalities of the app	Possible task
Automated counting	"How can you determine which number is displayed?"
It should be clear that the app itself counts, how many counters are in which column and also the total number of counters (if the corresponding settings are activated).	
Coloring when the bundling unit is exceeded	"When does the color change in the table header? And why?"
As soon as (in the decimal system) there are more than nine counters per column, the number of counters in the table head is colored red.	
Deleting of counters	"Remove all counters."
Erasing can be performed either by moving the counters upwards to the table header or by the shaking movements (see page. 6). However, it should be discussed with the class whether the shaking gesture is appropriate.	
Multitouch	"Create as quickly as possible the number 8."
It is possible to create counters simultaneously with several fingers.	
Automated bundling	"Place 16 counters in the ones column. What can you do now to get the usual representation of the number 16 in the place value chart?"
The pupils should recognize that moving to the left automatically bundles. At this point, the difference to the non-digital place value chart can be worked out.	
Bundling not possible	"Create the number 8, then move a counter from the ones column into the tens column. Explain what happens."
If there are fewer than ten counters (in the decimal system) in a column, they cannot be bundled by moving a counter to the left.	
Automated unbundling	"Create 2 tens and 3 ones. Then move one counter from the tens column to the ones column. Explain, what happens."
Analogous to the automated bundling, unbundling is to be investigated and explained.	
Constant value inspite of changes in the representation	"Create the number 23. What happens to the number when you move counters? Find different ways to display the number 23."
The pupils should now internalize the fact that bundling and unbundling change the representation of the number, but its value itself is preserved.	

Tasks combining the various functionalities could be as follows: "Set the number 58 only with ones. Then delete the number as quickly as possible."

Flexible understanding of place value

In addition to the standard representation of the number 21 in the place value chart as 2 tens and 1 one, other whole number representations are also possible - namely as 1 tens and 11 ones or also as 21 ones.



If a maximum of nine counters are used per place, it's called the **standard decomposition** of a number from which the usual number representation with the numbers 0 to 9 (in the decimal system) results. All other possible representations, on the other hand, are referred to as non-standard decompositions.

A **flexible understanding of place value** is the ability to **switch flexibly between the standard decomposition and non-standard decompositions of a number**.

Using the place value app, it is possible to learn the process of transition between standard decomposition and non-standard decompositions. The operations required for this step, such as bundling and unbundling or counting the counters, are taken over by the app, since they should already be highly developed by the child.

The necessity of gaining a flexible understanding of place value, for example, results from everyday language (e. g. "Felix Klein was born in 1849."), but is also found in mathematical contexts such as the half-written calculation methods of division (e. g. *1612 : 4*) or the written calculation methods of addition and subtraction (e. g. *137 + 346*).

Th	H	T	O
	18	4	9
	16		12
	4	7	13

The following types of tasks are suitable for the promotion of the flexible understanding of place value:

1. Write the following numbers!

   1**H** 32**T** 4**O**

   279**O**

   7**O** 31**T**

   3**H** 3**T** 14**O**

   Not all units are fully bundled, also the order of the values is not always the standard order. Therefore, the significance of the individual values must be recognized and the number determined.

2. Which number is larger?

   5**T** 3**O** or 4**T** 15**O**

   1**T** 14**O** or 2**T** 8**O**

   73**O** or 7**O** 3**T**

   4**T** 9**O** or 1**T** 29**O**

   The order and depth of the bundling are also varied here. Among other things, the idea is to be countered that larger digits (when neglecting the bundling unit) are given a larger value is to be countered.

3. Place the number 32 in the place value chart!

   Find another representation!

   How many different representations do you find?

   This sequence of tasks again illustrates the flexible representation possibilities of a number in the place value chart. It is possible to find rules for finding all representations, and the procedure can be used later on if the chart is expanded to the right (see page 22).

Automated bundling and unbundling, and thus the constant value of the displayed number when shifting counters, is the essential difference from a conventional place value chart. This also means that the pupils must have already internalized the bundling, so that it can be accepted here as an automated action.

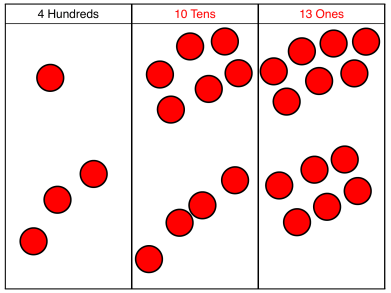
Written methods of addition

The written addition algorithm can be used to show how - starting from the ones – it is added place-wise and, if necessary, bundled. So that the counters can still be displayed clearly, when explaining this procedure with the digital place value chart, care must be taken that the numbers do not become too large.

It is not intended that the digital scoreboard be used as a tool for determining the results of any addition tasks. Rather, it is an aid to the **introduction of the method** in order to unite the written algorithm with the concept of place-wise adding and bundling. Therefore, a **parallel execution** of written addition and addition with the scoreboard is recommended.

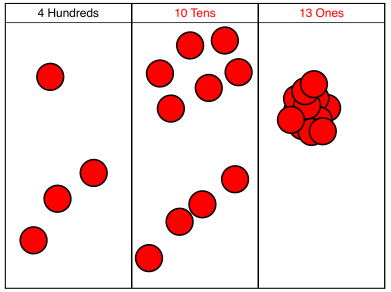
Recommended settings:
Show total off, so that the sum of the numbers is not shown.

1. Representing numbers




```
  1 6 7
+ 3 4 6
-------
```

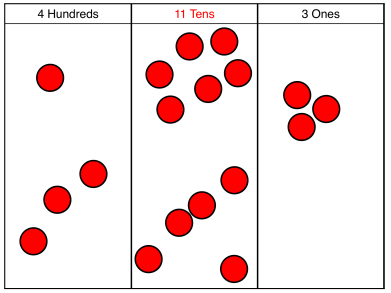
2. Adding the ones




```
  1 6 7
+ 3 4 6
-------
```

"13 ones"

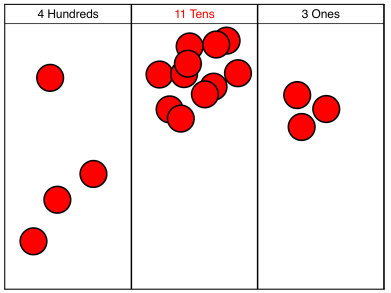
3. Bundling the ones




```
    1
  1 6 7
+ 3 4 6
-------
      3
```

"Write 3 (ones), memorize 1 (tens)"

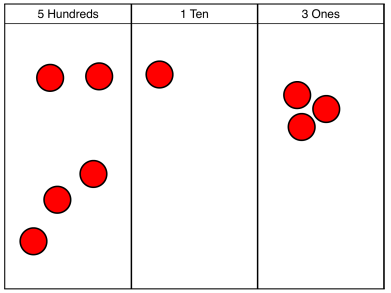
4. Adding the tens




```
    1
  1 6 7
+ 3 4 6
-------
      3
```

"11 tens"

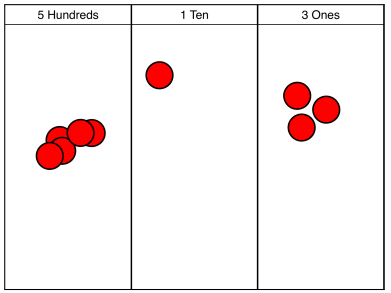
5. Bundling the tens




```
  1 1
  1 6 7
+ 3 4 6
-------
  1 3
```

"Write 1 (tens), memorize 1 (hundred)"

6. Adding the hundreds



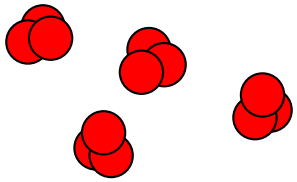

```
  1 1
  1 6 7
+ 3 4 6
-------
  5 1 3
```

"Write 5 (hundreds)"

In the classification of division situations, two types of division can be distinguished: partitive division and quotitive division. For the written method of division it is recommended to work with the concept of quotitive division. Using the example of the task 12:3, this means:

Quotitive division

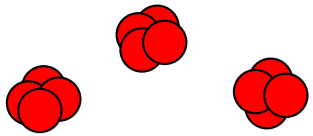
How many groups of 3 are there in 12?



groups of 3
How many groups?

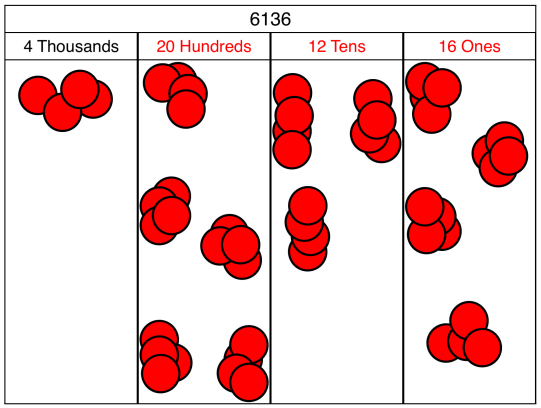
Partitive division

How much does everyone get when 12 are evenly distributed to 3 groups?



3 equal groups
How many in each group?

A possible approach using the example of the task 6136: 4 is shown on the next double page. In the final configuration, the image shown here is obtained.



It should be noted that the dividend 6136 is still present, but is now represented by groups of 4.

The step-by-step change in representation, beginning with the thousands, requires a flexible representation of the numbers, for which the digital place value chart is suitable. If one interprets the heaps of 4 as counting heaps, the result 1534 is shown in the place value chart. The numbers must be chosen in such a way that, on the one hand, the heaps do not become too large; on the other hand, the number of heaps should remain small (five), so that everything can still be well displayed. To generate tasks, it is also useful to multiply a result with small digits (e.g., 1534) with a relatively small number (e.g., 4).

The following tasks offer learning opportunities at different difficulty levels:

- Complete procedure feasible

  732 : 3 6216 : 4 7215 : 5

- Unbundling necessary (thousands)

  1365 : 3 2134 : 4

- Without remainders from the tens

  5728 : 4 6755 : 5

- Without remainders from the hundreds / only the hundreds

  4263 : 3 5648 : 4 7245 : 3 7284 : 6

- Digit zero
  (also with half-written methods solvable)

  4015 : 5 4506 : 3

- Divisions with remainders
  (suitable as a transition to decimal fractions)

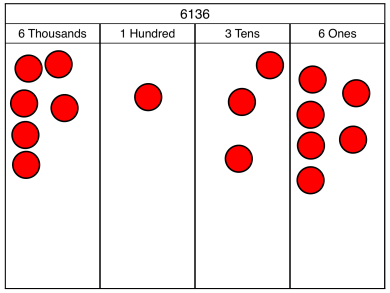
  8530 : 6

Recommended setting:
Show total on; This makes it clear that the dividend is not changed but it is only split in parts.

Written methods of division

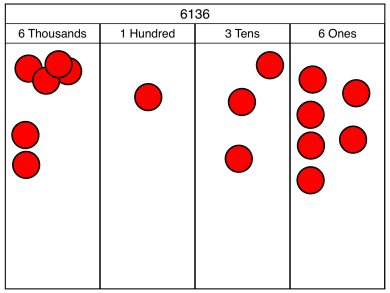
1. Representing numbers

$$6\ 1\ 3\ 6 : 4 =$$



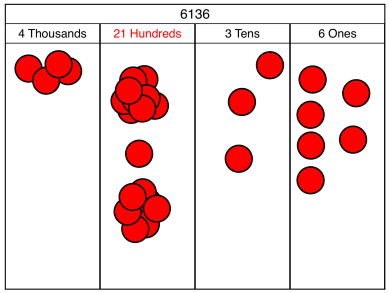
2. Splitting the thousands

$$\begin{array}{r} 6\ 1\ 3\ 6 : 4 = 1 \\ \underline{4} \\ 2 \end{array}$$



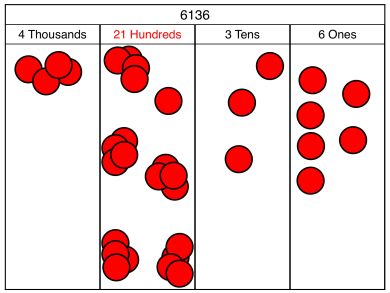
3. Unbundling the thousands

$$\begin{array}{r} 6\ 1\ 3\ 6 : 4 = 1 \\ \underline{4} \\ 2\ 1 \end{array}$$



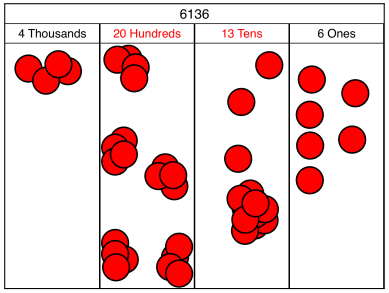
4. Splitting the hundreds

$$\begin{array}{r} 6\ 1\ 3\ 6 : 4 = 1\ 5 \\ \underline{4} \\ 2\ 1 \\ \underline{2\ 0} \\ 1 \end{array}$$



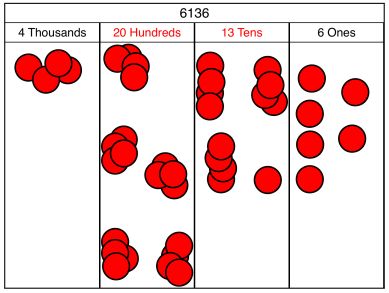
5. Unbundling the hundreds

$$\begin{array}{r} 6\ 1\ 3\ 6 : 4 = 1\ 5 \\ \underline{4} \\ 2\ 1 \\ \underline{2\ 0} \\ 1\ 3 \end{array}$$



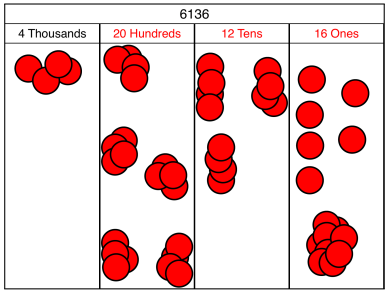
6. Splitting the tens

$$\begin{array}{r} 6\ 1\ 3\ 6 : 4 = 1\ 5\ 3 \\ \underline{4} \\ 2\ 1 \\ \underline{2\ 0} \\ 1\ 3 \\ \underline{1\ 2} \\ 1 \end{array}$$



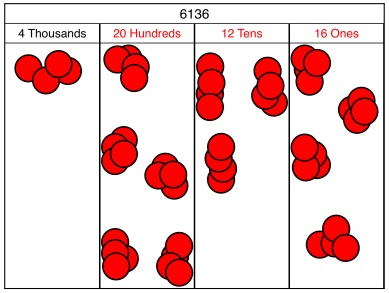
7. Unbundling the tens

$$\begin{array}{r} 6\ 1\ 3\ 6 : 4 = 1\ 5\ 3 \\ \underline{4} \\ 2\ 1 \\ \underline{2\ 0} \\ 1\ 3 \\ \underline{1\ 2} \\ 1\ 6 \\ 1\ 6 \end{array}$$



8. Splitting the ones

$$\begin{array}{r} 6\ 1\ 3\ 6 : 4 = 1\ 5\ 3\ 4 \\ \underline{4} \\ 2\ 1 \\ \underline{2\ 0} \\ 1\ 3 \\ \underline{1\ 2} \\ 1\ 6 \\ \underline{1\ 6} \\ 0 \end{array}$$



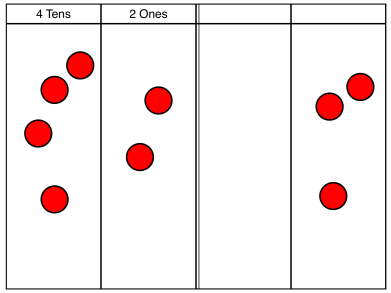
Decimal fractions

For the number range expansion from the whole numbers to the decimal fractions, it is useful to use the **permanence principle**: Previously known mathematical structures remain intact and are embedded in the extended number range.

A possible approach in teaching is as follows: After different representations for the number 32 have been found in the place value chart (see page 15, No. 3), this is also to be tried for the number 7. Since it has already been previously recognized that, from a standard decomposition, only unbundling to the right leads to further representations, there can only be a further representation for the number 7 if the scoreboard is extended to the right. This is possible through the app settings (see page 8).

For the new decimal places, the same laws of bundling and unbundling apply as for the whole numbers.

For the introduction, suitable settings can be used to initiate a discussion about naming the new place values.



Places: 4, Fractional Places: 2,
Show total: off, Headers: Integers

By turning off the setting “Show total”, the translation of the number representation in the place value chart into the corresponding decimal fraction can be learned .
It can also be done step-by-step by varying the setting “Headers” (see illustration). The “spell-out” function is useful for practicing the correct way of speaking (e.g. *“three point one two”* instead of *“three point twelve”*).

Even with decimal fractions, the flexible understanding of the place value can be trained analogously to the whole numbers (see page 15).

Recommended settings: Places 4, Fractional Places 2

1. Write thefollowing numbers!

   13**t**

   2**t** 14**h**

   1**T** 2**O** 7**h**

   3**O** 25**t**

2. Which number is larger?

   4**T** 9**O** or 1**T** 29**O**

   5**t** 3**h** or 4**t** 15**h**

   37**h** or 3**h** 7**t**

   2**O** 1**h** or 2**O** 1**t**

   5**T** 32**O** 13**h** or 8**T** 2 **O** 1**t** 3**h**

3. Create the number 2.35 in the place value chart!

   Find another representation!

   How many different representations do you find?

   Can there be more representations if the place value chart is differently structured? Explain!

So far, the project has not been used to test the app on decimal numbers. As soon as experiences are available, this guide can be adjusted accordingly.

About the project “Digitales Lernen Grundschule”

“Digital Lernen Grundschule” is a project of the Universities of Bremen, Hamburg and Potsdam as well as the Universities of Education of Ludwigsburg and Schwäbisch Gmünd and the Ludwig-Maximilians-Universität München, made possible by the Deutsche Telekom Foundation.

Every university develops concepts for the use of digital media in elementary education. At the University of Potsdam, the development of concepts is based on three guidelines:

Linking real and virtual spaces of action

Real experiences must not be destroyed by the use of digital technologies, but at the same time the new possibilities should be used sensibly. In all concepts, therefore, a parallel approach of real and virtual spaces of action is provided in order to establish a link between the two worlds and to exploit the respective advantages of each.

Connectivity to secondary school

Specific basic concepts will be selected that take up the previous experience of the children, develop them and consolidate them into further usable basic concepts for learning in the secondary levels. The ideas behind the respective concepts must be transferable to similar or later learning contents.

Based on theory of subject-specific didactic

In addition to media competency, theelementary school primarily imparts content-related and process-related subject-specific competencies. In the project, the competency of subject-specific didactic of the researchers at the University of Potsdam is used to elaborate the content of digital tools.

According to these guidelines, seven teaching concepts are developed and evaluated at the University of Potsdam:

Numerical understanding A digital place value chart enables a flexible understanding of the place value.

Spatial relationships Digital media help to mathematically describe real geometrical configurations.

Reading fluency By reading at the same time the audio book is played, the reading fluency improves.

Analogy formation The parallel execution of real experiments and virtual analogies supports the process of comprehension.

Data logging The digital detection and evaluation of measured data supports the implementation of real experiments.

Loops and structures First programming experiences with actual cubes are made.

Algorithms in everyday life On the basis of everyday phenomena, fundamental principles of algorithmic thinking are developed.

As a result, the project will provide ready-to-use and directly applicable concepts, as well as explicate a common theoretical basis for the concepts.

In particular, the integration into the school-practical studies, the placing of master’s theses, and a cross-disciplinary colloquium ensure the practical and theorized integration into the Studiey courses for future teachers at the University of Potsdam.

Notes